Modeling spectral filtering effects on color-matching functions:

Implications for observer variability

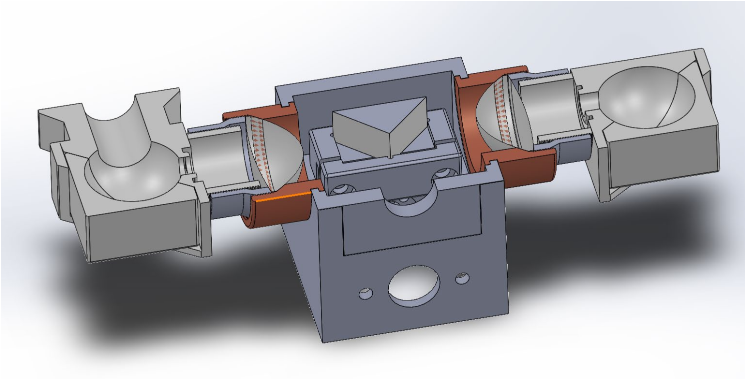
LUVIN M RAGOO

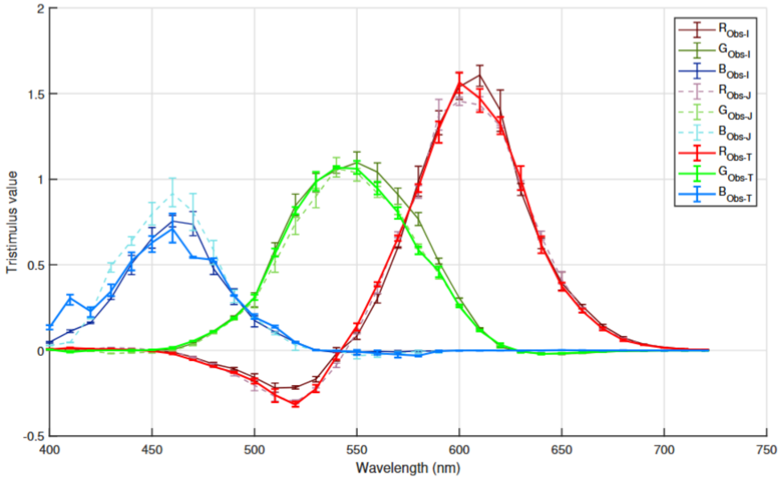
luvin.m.ragoo@ntnu.no

Motivation

Colourlab

You may recall...





- An earlier work where attempts to measure individual CMFs were made
- Some promising results, but...
- Experiments were still lengthy (15 – 20 hours)
- Is there a way to *summarise* individual differences in CMFs?

Ragoo, Luvin, Ivar Farup, and Jan Henrik Wold. "Apparatus and Method for Measuring Individual Colour-Matching Functions." (2024).

An idea...

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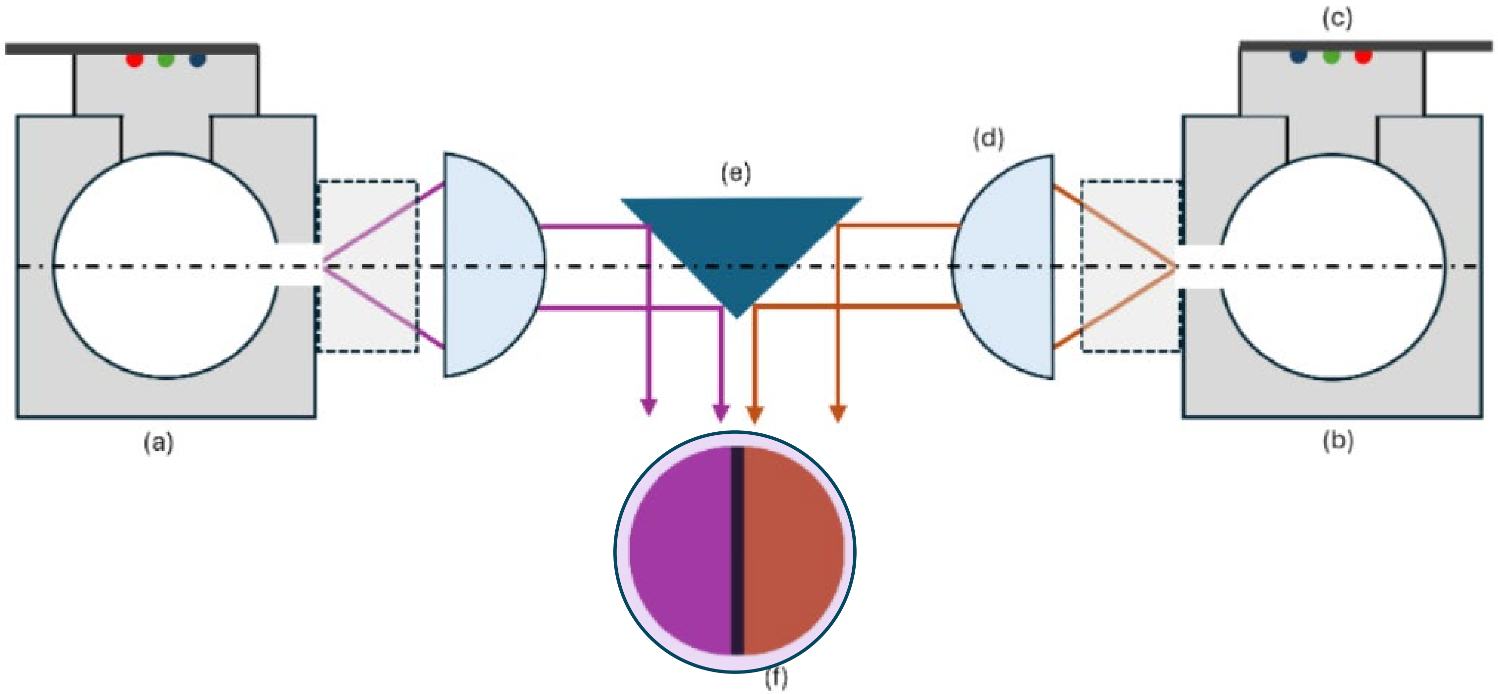

Fig. 1. Schematic diagram of the colorimeter. (a) Integrating chamber for the test field, it has two inputs (not shown on the diagram), the near-monochromatic test stimuli and the LED light engine. (b) The integrating chamber for the match field has only one input for the LED light engine. (c) LED light engine. (d) Aspheric condenser lens. (e) Knife-edge right-angle prism mirror. (f) Resulting bipartite field with adjustable gap width.

Ragoo, Luvin, Ivar Farup, and Jan Henrik Wold. "Apparatus and Method for Measuring Individual Colour-Matching Functions." (2024).

Experiment
Primaries and test lights

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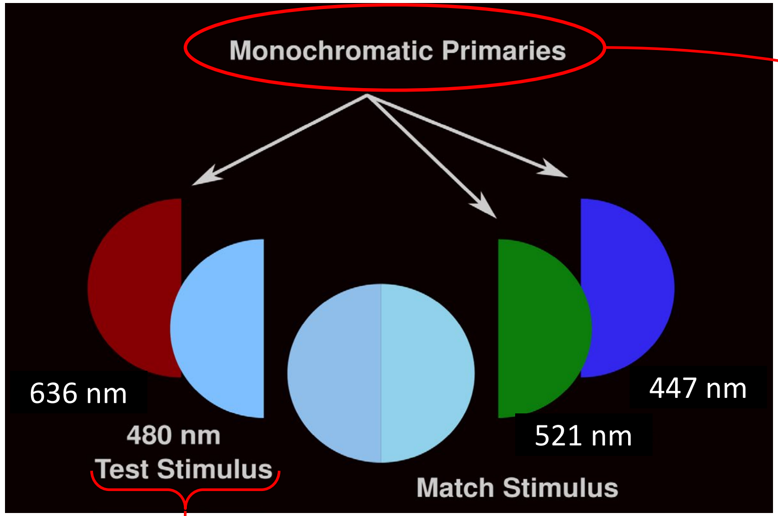


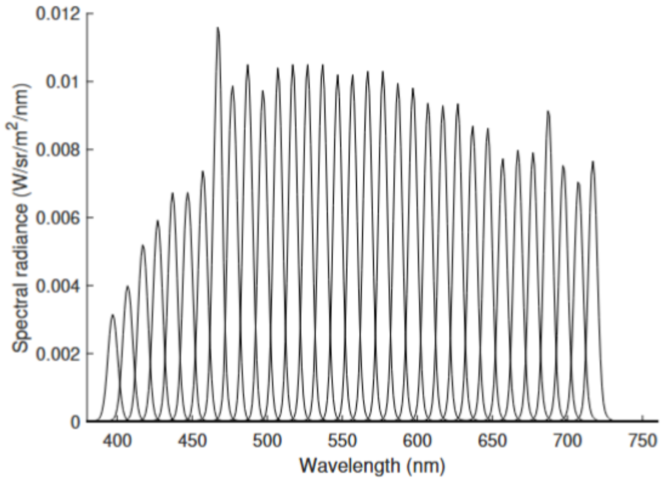


Figure 4. SPDs of the test lights

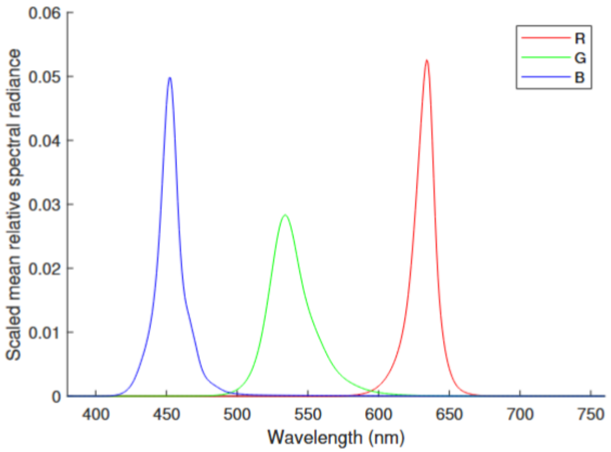
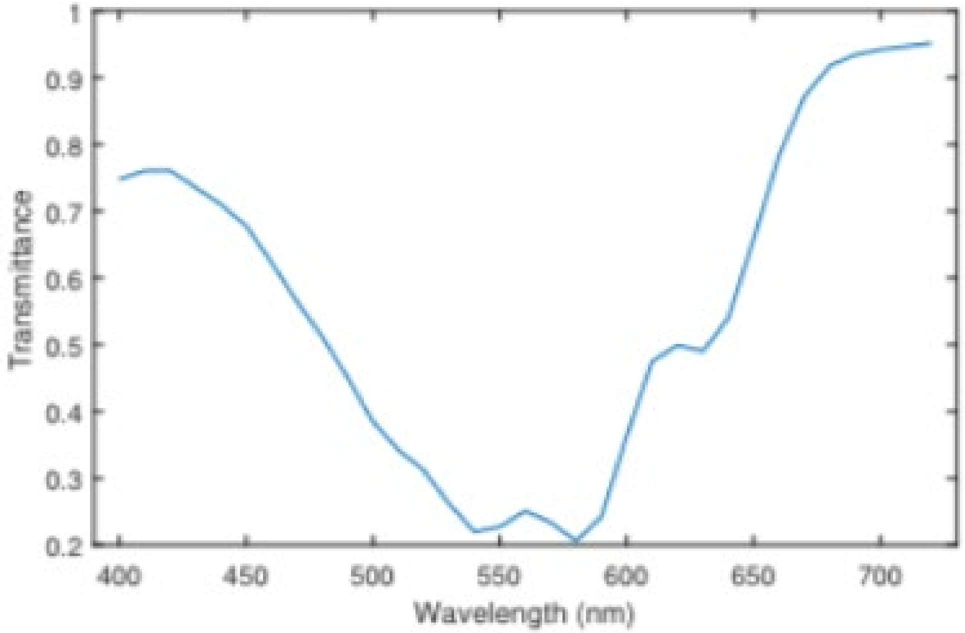


Figure 7. Scaled mean relative SPDs of the RGB LED lights, the scaling making their integrals (i.e. sums of their values at the sampling wavelengths of the spectroradiometer) all equal to 1 (The SPDs define the reference colour stimuli, **R**, **G**, and **B**.)

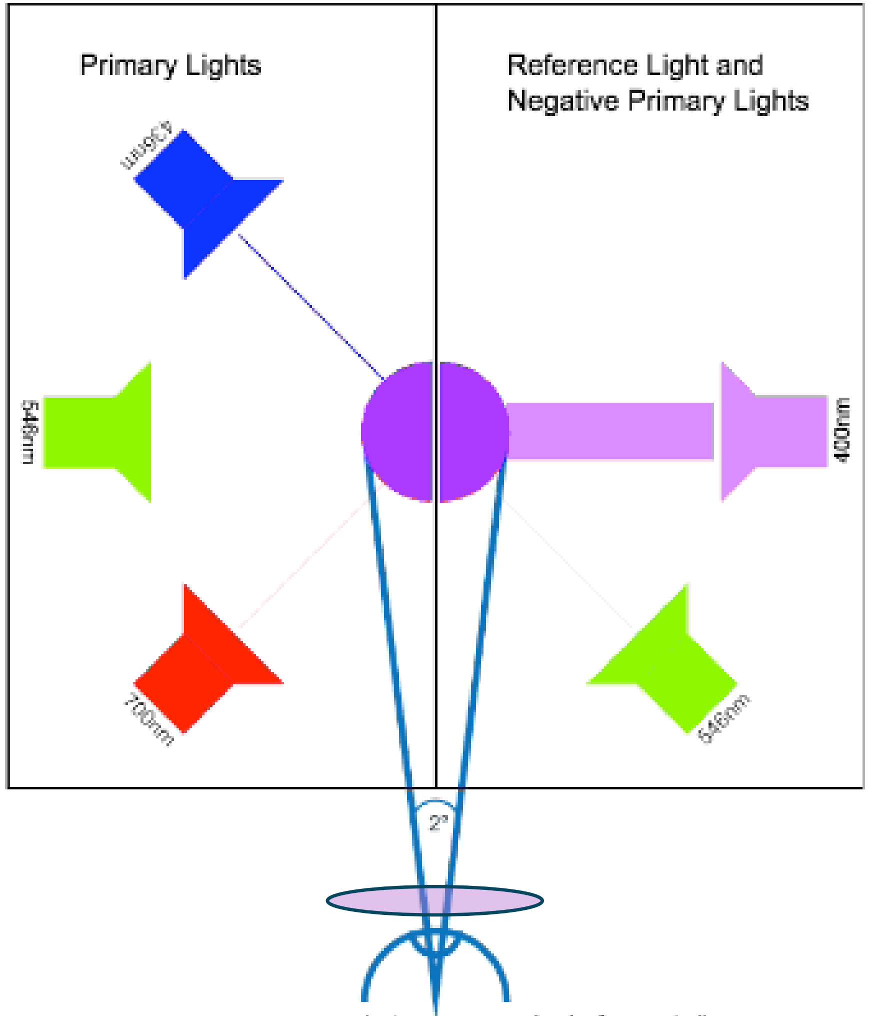
Ragoo, Luvin, Ivar Farup, and Jan Henrik Wold. "Apparatus and Method for Measuring Individual Colour-Matching Functions." (2024).

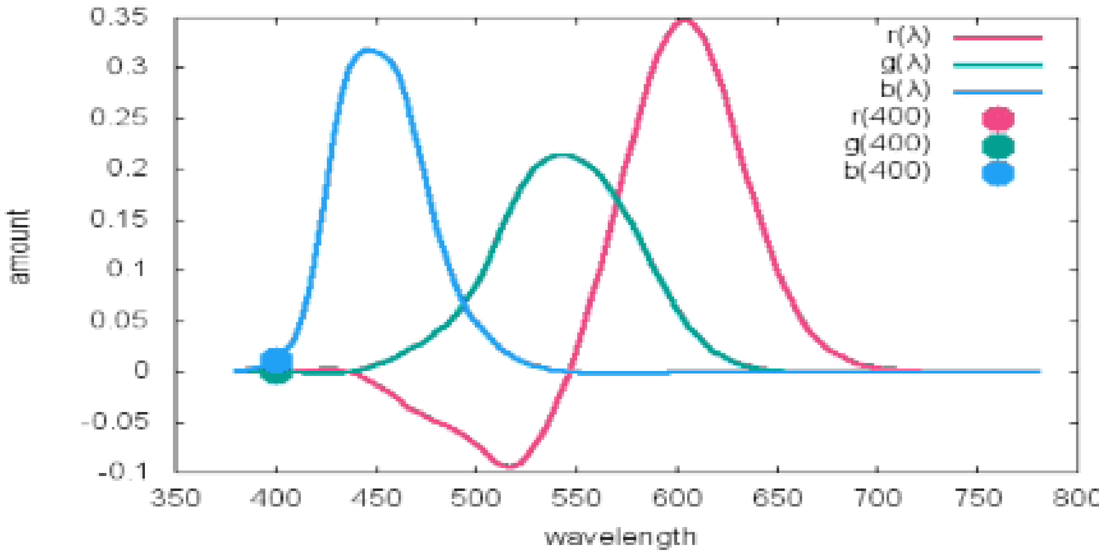
Experiment
Filter transmittance

Ground Truth Filter transmittance (resampled):



Experiment
Method





Reference : A Beginner's Guide to (CIE) Colorimetry | by Chandler Abraham | Color and Imaging | Medium

Experiment
Measurement

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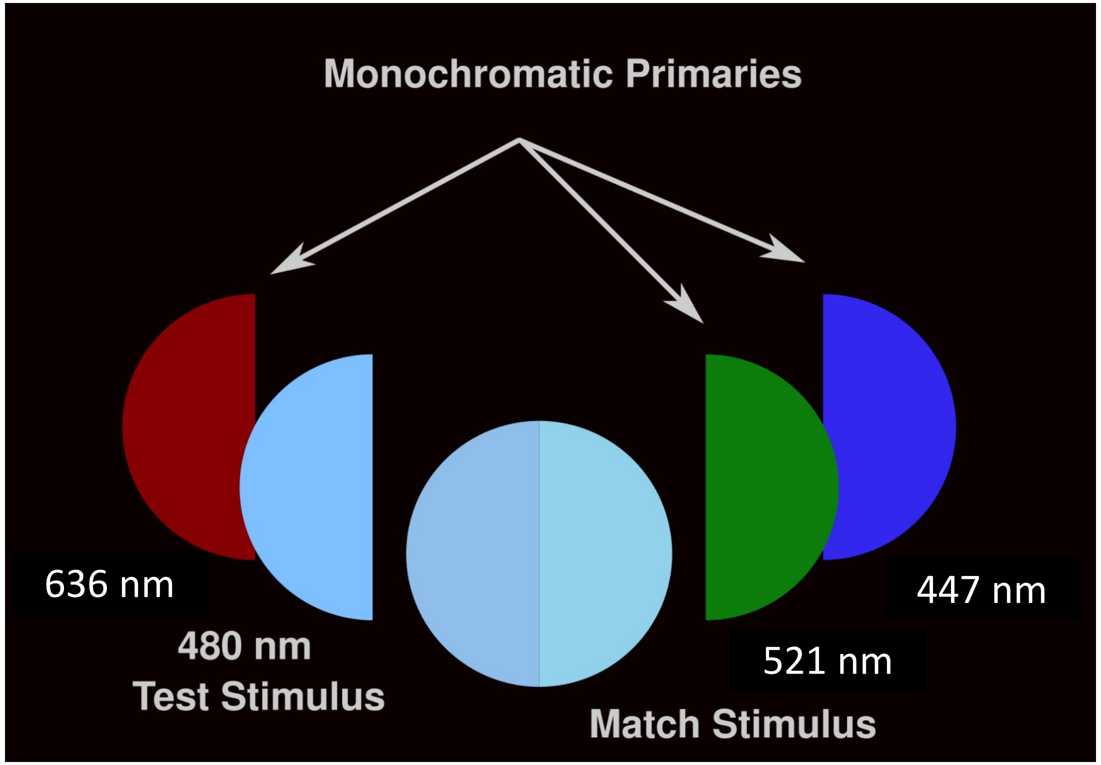
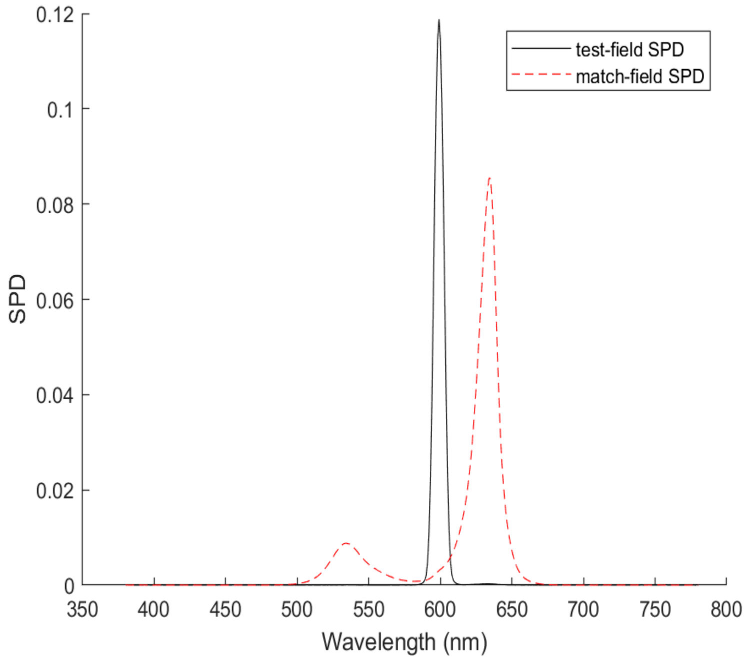


By Maneesh - Own work, CC BY-SA 4.0,
https://commons.wikimedia.org/w/index.php?curid=105220816

For example, for a test light of peak wavelength, 600 nm :



Experiment
Computing colour-matching coefficients

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$$\mathbf{A}_{\lambda_i} = \begin{bmatrix} \mathbf{R} & \mathbf{G} & \mathbf{B} & \bar{\mathbf{S}}_{\lambda_i} \end{bmatrix}$$

$$\mathbf{A}_{\lambda_i}\mathbf{x}_{\lambda_i} = \mathbf{b}_{\lambda_i}$$

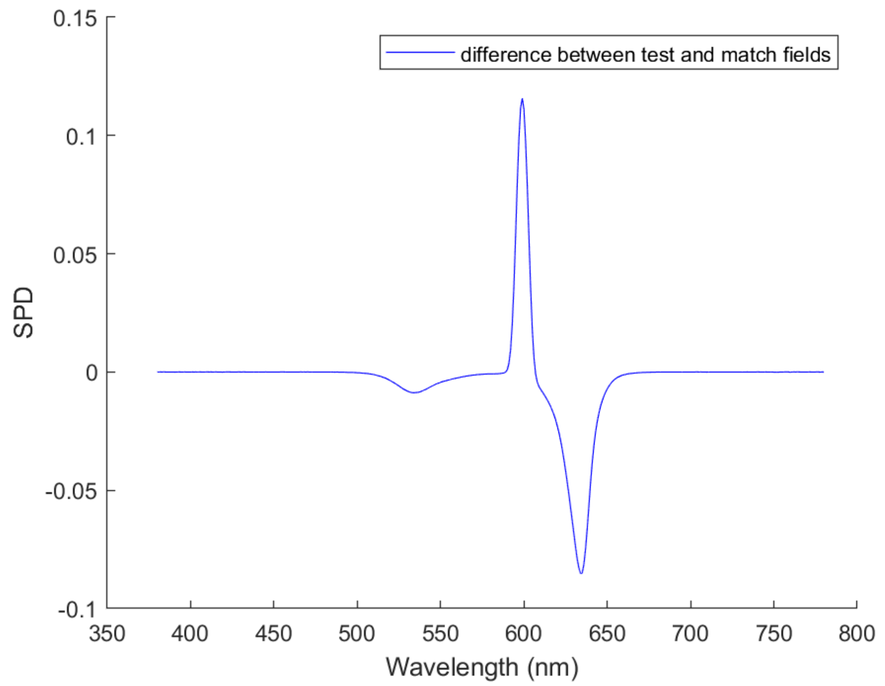
$$\mathbf{b}_{\lambda_i} = \bar{\mathbf{T}}_{\lambda_i} - \bar{\mathbf{M}}_{\lambda_i}$$

$\mathbf{R}, \mathbf{G}, \mathbf{B}$ *and* $\mathbf{S}_{\lambda_i}$ *are 401 by 1 vectors representing the spectral power distribution of the Red, Green, Blue primaries and the spectral light at peak wavelength* λ_i

$\mathbf{b}_{\lambda_i}$ is a 401 by 1 vector representing the difference between the match and test field.

Finally, the coefficients for the primaries that would match a test stimulus with peak wavelength λ_i , can be obtained:

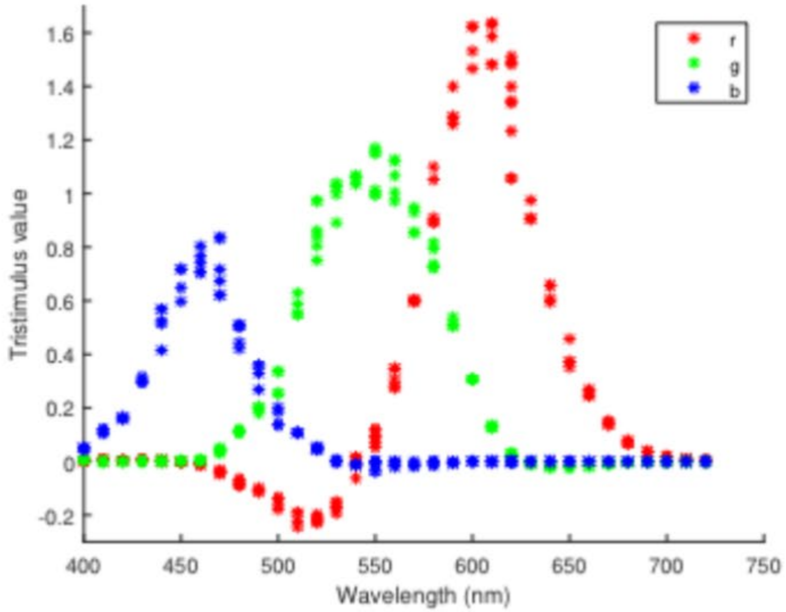
$$\mathbf{x}_{\lambda_i} = (\mathbf{A}_{\lambda_i}^T \mathbf{A}_{\lambda_i})^{-1} \mathbf{A}_{\lambda_i}^T \mathbf{b}_{\lambda_i}$$



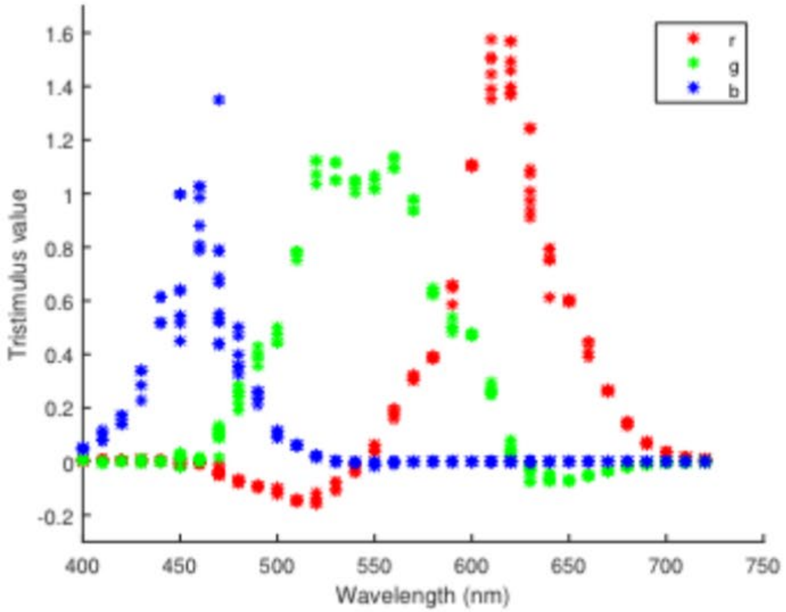
Results

Two distinct sets of tristimulus values (CMFs)

For the same observer :



(a) without filter



(b) with filter

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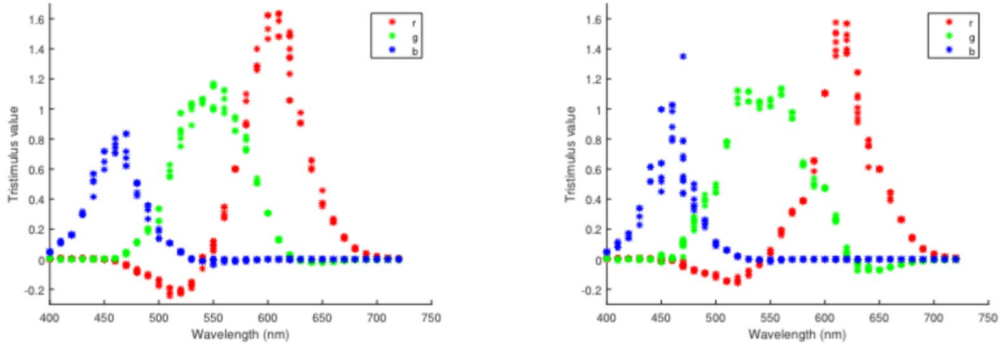
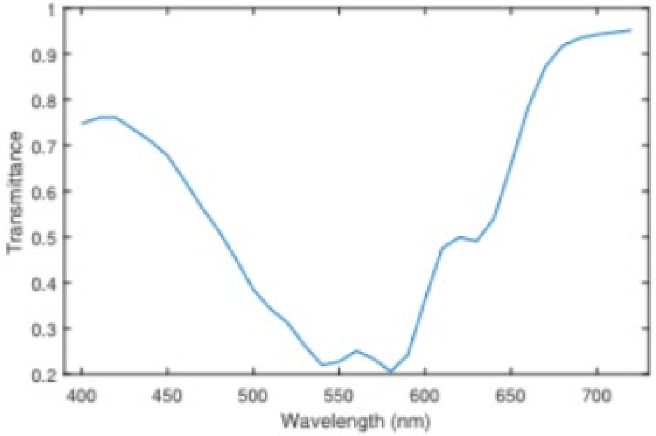
Recovering filter transmittance



(a) without filter (b) with filter

Ground Truth Filter transmittance



From these two related sets of CMFs, can we estimate the filter transmittance?

$$\mathbf{C}_1 = \mathrm{diag}(\mathbf{f})\mathbf{C}_2\mathbf{M}^{-1} \qquad (1)$$

where:

- $\mathbf{C}_1, \mathbf{C}_2 \in \mathbb{R}^{N\times 3}$: CMF matrices at 10 nm intervals (400-720nm) representing the target and source CMFs respectively.
- $\mathbf{M} \in \mathbb{R}^{3\times 3}$: Transformation matrix
- $\mathbf{f} \in \mathbb{R}^{N}$: Spectral transmittance vector (corresponding to "purple" filter's transmittance in experiment)

Can be re-written as:

$$\mathrm{argmin}_{\mathbf{M},\mathbf{f}} \, \|\mathbf{C}_1\mathbf{M} - \mathrm{diag}(\mathbf{F})\mathbf{C}_2\|^2 + \alpha\,|\mathbf{Tf}|^2$$

Regularisation term

N Filter estimates

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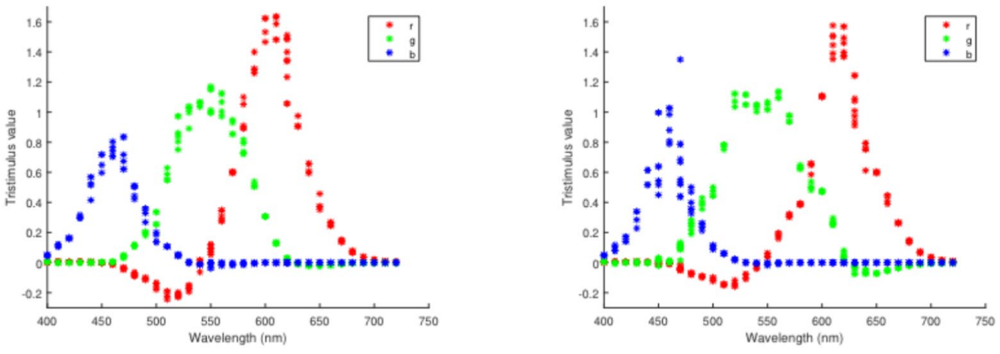


(a) without filter

(b) with filter

Given, the several experimental tri-stimulus values, we can compute several plausible filter estimates:

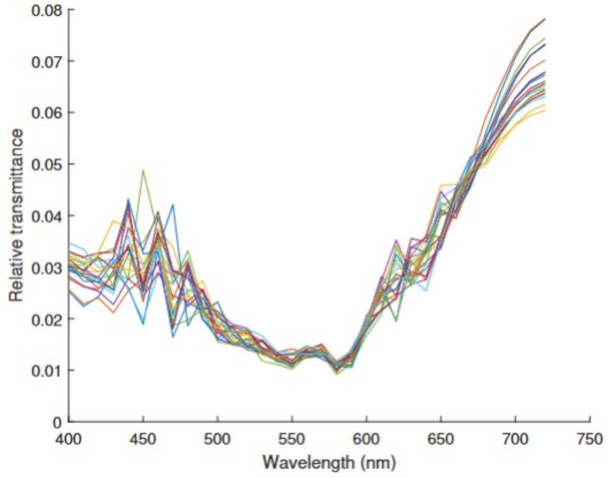
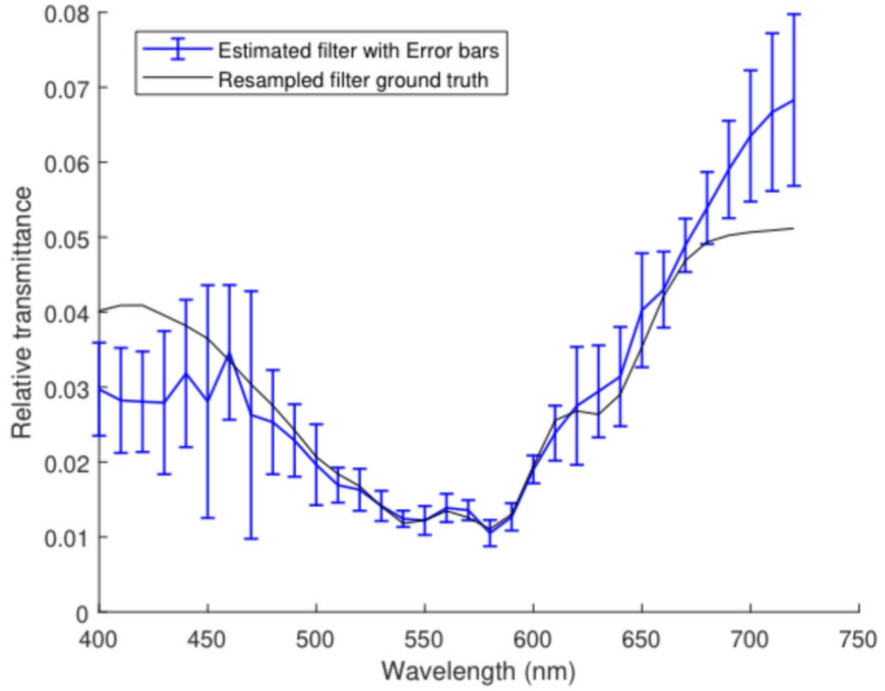


Fig. 4. The first 20 of N filter estimates



Discussion

Extending to two unrelated CMFs

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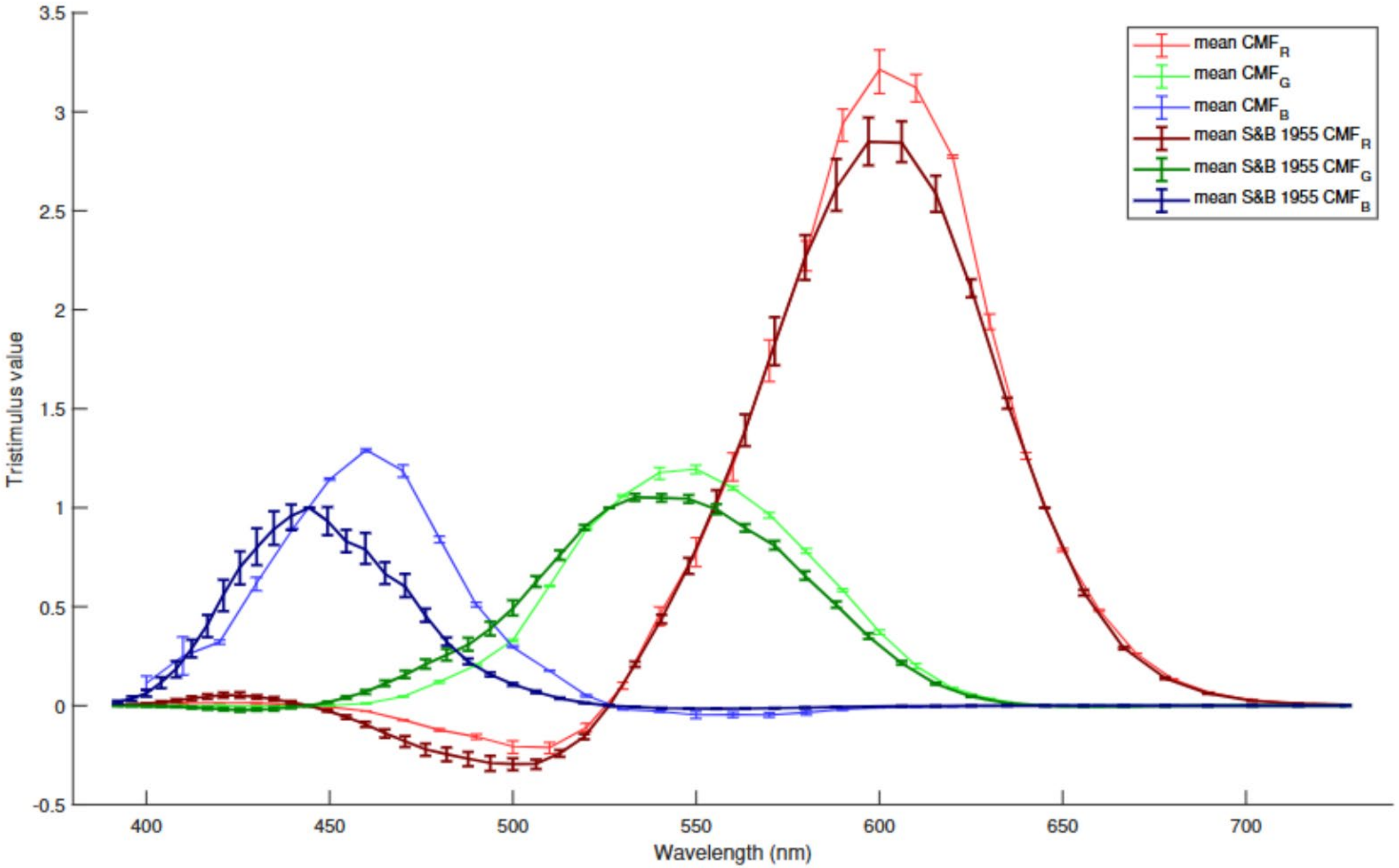
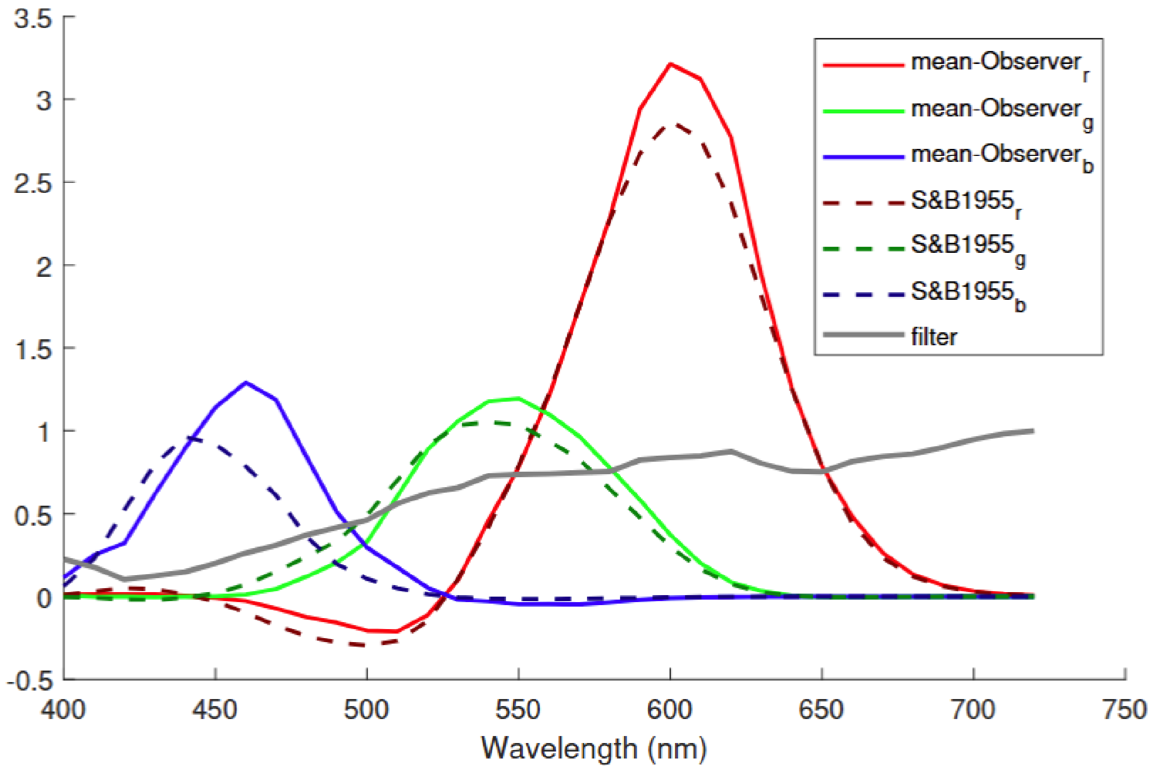
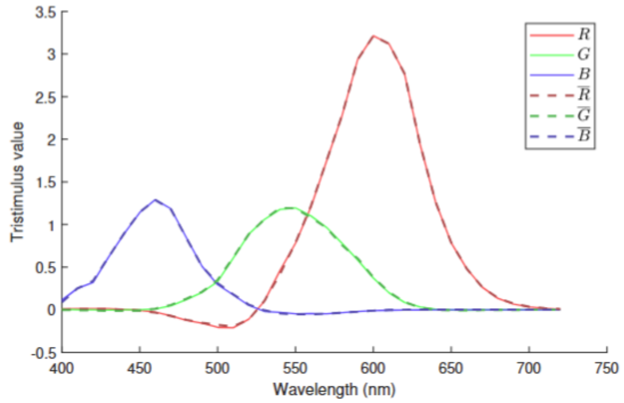


Discussion

Extending to two unrelated CMFs



The spectral filter that would transform from the Stiles and Burch 1955 CMFs to ours, is a yellow filter!



L. M. Ragoo, I. Farup, G. Finlayson and C. F. Andersen, ‘Modeling spectral filtering effects on color-matching functions: Implications forobserver variability,’ submitted to Optics Express, 2025

Discussion

Extending to two unrelated CMFs

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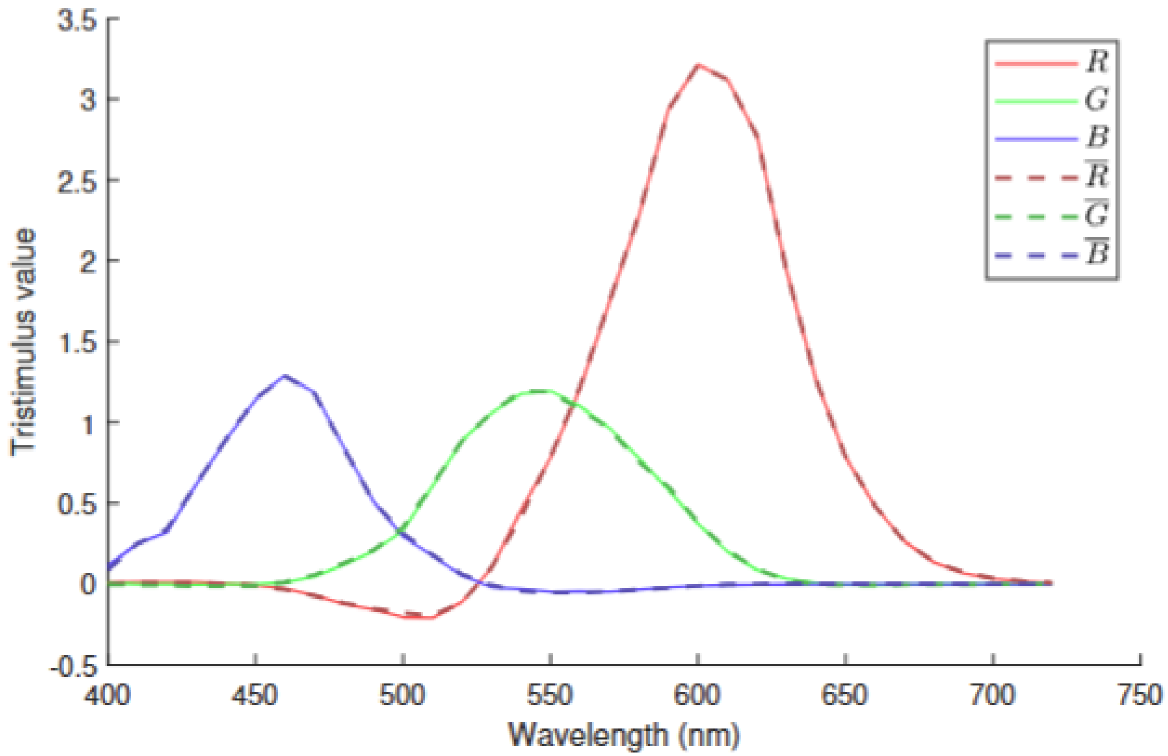
As a validation of the computed filter :



$$\overline{\mathbf{C}}_{\mathrm{ICVIO}} = \mathrm{diag}(\mathbf{f}_{\mathrm{ICVIO}})\mathbf{C}_{\mathrm{SB1955}}\mathbf{M}^{-1}_{\mathrm{ICVIO}} \quad (13)$$

where :

- $\overline{\mathbf{C}}_{\mathrm{ICVIO}}$ are the reconstructed ICVIO 2° mean observer CMFs.
- $\mathbf{f}_{\mathrm{ICVIO}}$ is the computed spectral filter that would convert from the Stiles and Burch 1955 2° mean observer CMFs to that of ICVIO's
- $\mathbf{C}_{\mathrm{SB1955}}$ are the Stiles and Burch 1955 2° mean observer CMFs
- $\mathbf{M}_{\mathrm{ICVIO}}$ is the computed 3×3 linear transformation matrix

- Computed filter models the difference between our two CMFs dataset rather well.
- It explains some of the physiological differences that may exist. For eg, yellowing of the crystalline lens.
- Important to note that some sources of physiological differences may NOT be effectively modeled, such as genotopic variations in the cone photopigments.

Discussion

Implications for modelling observer variability

- If we start from a standard observer CMFs as reference, individual variations in CMFs can be represented as a Nx1 filter, that would transform the reference CMFs to that of the individual.

- As a result, significant dimensionality reduction is achieved. Instead of 3xN functions, individual differences in CMFs can be represented by a single filter.

- Potential for reducing experimental times significantly.

luvin.m.ragoo@ntnu.no

Summary

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Given two sets of related CMFs, the filter that would transform between them can be estimated.

This model can be extended to unrelated CMFs sets where the differences between them can be expressed in terms of a spectral filter.

Observer variability in CMFs could potentially be modelled by a spectral filter

Future work : Investigate in shorter experiments to characterise individual filters, rather than CMFs.

Modeling spectral filtering effects on Colour-Matching Functions: Implications for observer variability

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